Asymptotic Average Shadowing Property

A review of the results published in:

"The asymptotic average shadowing property and transitivity",
Gu Rongbao, Nonlinear Analysis 67 (2007) 1680–1689

Assume that X is a compact metric space and $f : X \to X$ is a continuous map.

Definition. A sequence $\{x_i\}_{i=0}^{\infty}$ of points from X is an asymptotic-average pseudo orbit of f if $\lim_{n\to\infty} 1/n \cdot \Sigma_{i=0}^{n-1} d(f(x_i), x_{i+1}) = 0$.

Definition. A sequence $\{x_i\}_{i=0}^{\infty}$ of points from X is asymptotically shadowed in average by a point $z \in X$ if $\lim_{n\to\infty} 1/n \cdot \Sigma_{i=0}^{n-1} d(f^i(z), x_i) = 0$.

Definition. The map f has the asymptotic average shadowing property (AASP) if every asymptotic-average pseudo orbit of f is asymptotically shadowed in average by some point $z \in X$.

Theorem. If f has AASP then f^k has AASP for every $k > 1$.
If f^k has AASP for some $k > 1$ then f has AASP.

Theorem. If f is a surjection with AASP then f is chain-transitive (i.e. for any $\delta > 0$ any two points $a, b \in X$ can be connected by a finite δ-pseudo orbit of f starting at a and terminating at b).

Theorem. If f is a L-hyperbolic homeomorphism with AASP then f is topologically transitive.

(Presented by Marcin Kulczycki)