SEMINARIUM UKŁADY DYNAMICZNE

Tytuł: **Coupled Expanding maps (wg wspólnego artykułu z P. Oprochą)**
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Data: **18 I 2013**

Zreferowane zostały następujące twierdzenia:

Theorem 1. Let (X, d) be a complete metric space and let $f\colon X \to X$. Let $m \geqslant 1$ and let A be an $m \times m$ transition matrix. Assume that A is irreducible, but not a cyclic permutation (i.e. there is a row of A with sum of its entries at least 2). Let f be strictly A-coupled-expanding in the family of closed and bounded sets $V_1, \ldots, V_m \subset X$ and let f be continuous on $\bigcup_{i=1}^m V_i$. Assume additionally that for every $\varepsilon > 0$ there is $\delta > 0$ such that for every set $B \subset \bigcup_{i=1}^m V_i$ with $\operatorname{diam}(B) < \delta$ the inequality $\operatorname{diam}(f^{-1}(B) \cap V_j) < \varepsilon$ holds for $j = 1, \ldots, m$. If there exists $\kappa \in \Sigma_A$ such that $\lim_{n\to\infty} \operatorname{diam}(V_\kappa^n) = 0$, then there exists a perfect and f-invariant set $\Lambda \subset \bigcup_{i=1}^m V_i$ such that:

(1) The map $f|_\Lambda$ is chaotic in the sense of Auslander & Yorke.
(2) There is $\varepsilon > 0$ and a Mycielski ε-scrambled set $M \subset \Lambda$ such that $\overline{\bigcup_{i=0}^t f^i(M)} = \Lambda$ for some integer $t \geqslant 1$. In particular $f|_\Lambda$ is ε-chaotic in the sense of Li & Yorke.
(3) If A is primitive, then $f|_\Lambda$ is mixing and M is dense in Λ.
(4) There exists a continuous map $\pi\colon \Lambda \to \Sigma_A$ with dense range such that $\pi \circ f = \sigma \circ \pi$.

Theorem 2. Let (X, d) be a complete metric space and let $f\colon X \to X$. Let $m \geqslant 1$ and let A be an $m \times m$ transition matrix. Assume that A is irreducible, but not a cyclic permutation. Let f be strictly A-coupled-expanding in the family of closed and bounded sets $V_1, \ldots, V_m \subset X$ and let f be continuous on $\bigcup_{i=1}^m V_i$. Assume that there are constants $\mu_1, \ldots, \mu_m > 0$ such that $d(f(x), f(y)) \geqslant \mu_i d(x, y)$ for every $x, y \in V_i$, $i \in \{1, \ldots, m\}$. Assume additionally that there is $k \geqslant 1$ and $u \in \Sigma_A$ such that $u[1] = u[k+1]$ and $\mu_{u[1]} \cdot \ldots \cdot \mu_{u[k]} > 1$. Then:

(1) There exists a closed and f-invariant set $\Lambda \subset \bigcup_{i=1}^m V_i$ such that $f|_\Lambda$ is chaotic in the sense of Devaney and there is a continuous map $\pi\colon \Lambda \to \Sigma_A$ with dense range such that $\sigma \circ \pi = \pi \circ f$.
(2) There exist an irreducible transition matrix D, which is not a cyclic permutation, and a compact and f-invariant set $\Gamma \subset \Lambda$ such that $\pi|_\Gamma$ is a homeomorphism and $(\Gamma, f|_\Gamma)$ is conjugate to (Σ_D, σ) (which means that dynamical properties of $f|_\Gamma$ are exactly the same as those of σ on Σ_D).
(3) If A is primitive, then Γ can be chosen so that $f|_\Gamma$ is topologically mixing (and therefore D is primitive).